Experiences in Developing and Using an Eigenvector Implementation of a Dynamic Terrestrial Food-chain Model

International MACCS User Group (IMUG), 20-22 September 2021

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Contents

Background

Background

- For current UK new build programmes, we have supported the Level 3 PSAs for the applicants
- Also transboundary assessments under Article 37 of the Euratom Treaty
  - Requires expected levels of contamination of foodstuffs
  - None of the codes we had at the time (PC COSYMA had been used for the Level 3 PSA) gave this an output
  - Investigated alternative approaches

Food-chain models

Food-chain and ingestion models

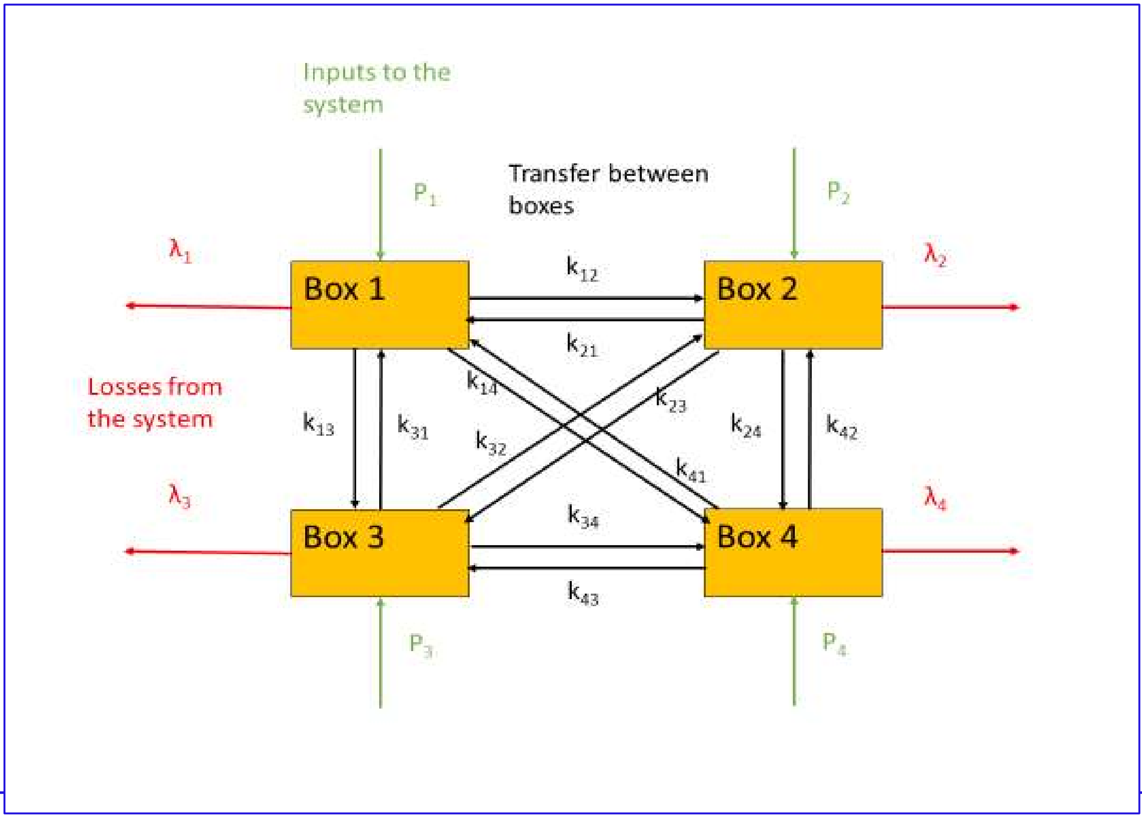
- Generic Types of Model

  - Multiplicative models
    - Uses a series of factors to relate levels of radioactivity in the various compartments to people, for example
      - [activity ingested] = k_1 x [deposited activity] x k_2 x[activity in plant/animal] x = k_3 x [activity in food]
      - Where there is a database of values for the transfer factors (k_n) for nuclide and each transfer
    - Models are simple but give little information on the time dependence

  - Dynamic box models
    - Progress of radioactivity through the food chain modelled by a series of interconnected compartments
    - Detail of model is determined by number of compartments to reflect complexity needed and information available to validate model
    - Models time dependence of activity in each compartment

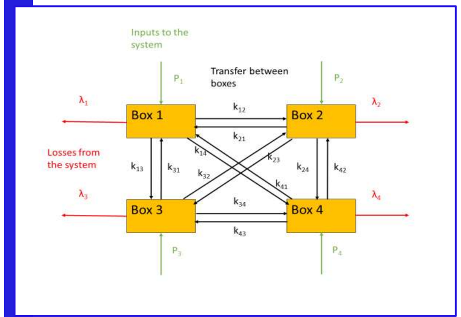
General scheme of the dynamic box model

- The boxes can represent environmental, plant, or animal compartments
- Concentration assumed to be homogenous in each compartment
- Transfer and loss is assumed to be a first order process where the rate is the transfer constant multiplied by the inventory in the source box



Solving the Box Model problem

- The Box Model problem reduces to a set of differential equations
- For Box 1 in previous figure, the activity in that compartment (X_1) is given by the following differential equation



$$\frac{\partial X_1}{\partial t} = -\lambda_1 X_1 - k_{12} X_1 - k_{13} X_1 - k_{14} X_1 + k_{21} X_2 + k_{31} X_3 + k_{41} X_4$$

$$\frac{\partial X_i}{\partial t} = -\lambda_i X_i - \left(\sum_{j=1}^{n\ (j \neq i)} k_{ij} \right) X_i + \sum_{j=1}^{n\ (j \neq i)} k_{ji}\ X_j$$

- This set of *n* differential equations (one for each box) can be solved in a number of ways
  - Numerical integration
  - Writing the equation in matrix form and posing as an Eigenvalue problem

$$\frac{\partial \underline{X}}{\partial t} = A\ \underline{X}$$

Solving the Box Model problem by finding the Eigenvalues and Eigenvectors

- For the square *n* x *n* matrix *A*, there will be *n* Eigenvectors such that

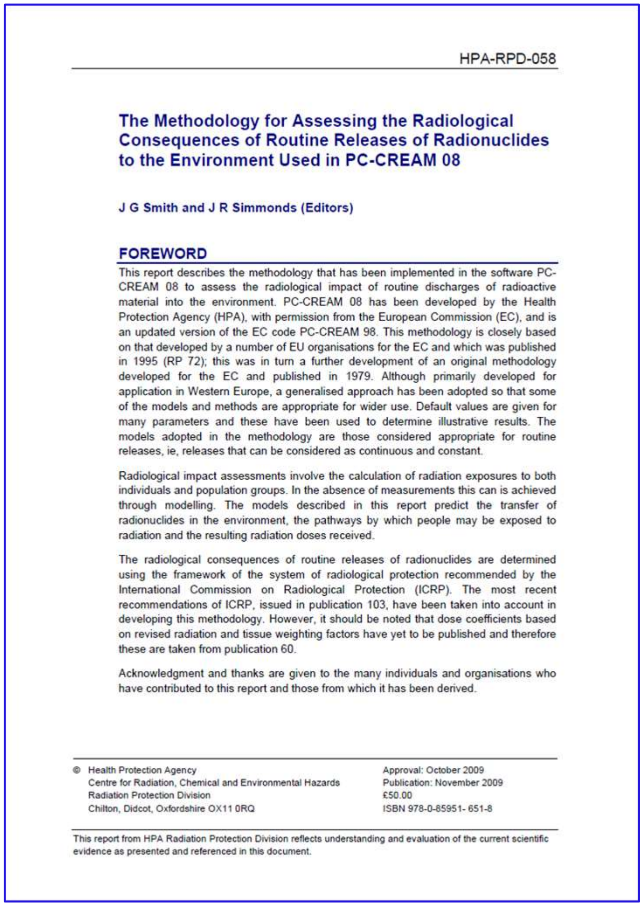
$$A\,\underline{X_e} = \mu\,\underline{X_e}$$

- Where μ is a scaler – the Eigenvalue
- The time-dependent functions for activity in each box will then be given by

$$X_i(t) = \sum_{j=1}^{n} C_{0j}\, e^{\mu_j t} EV_{ij}$$

- Where C_{0j} is the *j*th component of a vector of constants determined by the initial conditions
- EV_{ij} is a matrix of the Eigenvectors
- μ_j is the *j*th Eigenvalue
- A software package like Mathcad for example can be used to determine the Eigenvalues, Eigenvectors, and constants for the problem matrix and the initial conditions
  - https://www.mathcad.com/en

Example Food-chain Box Model - FARMLAND



HPA-RPD-058

The Methodology for Assessing the Radiological Consequences of Routine Releases of Radionuclides to the Environment Used in PC-CREAM 08

J G Smith and J R Simmonds (Editors)

FOREWORD

This report describes the methodology that has been implemented in the software PC-CREAM 08 to assess the radiological impact of routine discharges of radioactive material into the environment. PC-CREAM 08 has been developed by the Health Protection Agency (HPA), with permission from the European Commission (EC), and is an updated version of the EC code PC-CREAM 98. This methodology is closely based on that developed by a number of EU organisations for the EC and which was published in 1995 (RP 72); this was in turn a further development of an original methodology developed for the EC and published in 1979. Although primarily developed for application in Western Europe, a generalised approach has been adopted so that some of the models and methods are appropriate for wider use. Default values are given for many parameters and these have been used to determine illustrative results. The models adopted in the methodology are those considered appropriate for routine releases, ie, releases that can be considered as continuous and constant.

Radiological impact assessments involve the calculation of radiation exposures to both individuals and population groups. In the absence of measurements this can is achieved through modelling. The models described in this report predict the transfer of radionuclides in the environment, the pathways by which people may be exposed to radiation and the resulting radiation doses received.

The radiological consequences of routine releases of radionuclides are determined using the framework of the system of radiological protection recommended by the International Commission on Radiological Protection (ICRP). The most recent recommendations of ICRP, issued in publication 103, have been taken into account in developing this methodology. However, it should be noted that dose coefficients based on revised radiation and tissue weighting factors have yet to be published and therefore these are taken from publication 60.

Acknowledgment and thanks are given to the many individuals and organisations who have contributed to this report and those from which it has been derived.

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This report from HPA Radiation Protection Division reflects understanding and evaluation of the current scientific evidence as presented and referenced in this document.

- FARMLAND was developed by NRPB (now Public Health England) in 1990s
- Model is used in:
  - PC COSYMA
  - PACE
  - PC-CREAM
- PC-CREAM methodology report Appendixes A and B describe the FARMLAND model
  - The box model schemes
  - The transfer coefficients

https://assets.publishing.service.gov.uk/government/uploads/system/uploads/attachment_data/file/434637/HPA-RPD-058_June_2015.pdf

FARMLAND Foodchain models (in PC-CREAM)

- Green vegetable
- Grain
- Potato
- Fruit

- Cow (milk, milk products, meat) + undisturbed pasture model
  - Iodine
  - Caesium
  - Strontium
  - Non actinides

- Sheep (meat) + undisturbed pasture model
  - Iodine
  - Caesium
  - Strontium
  - Non actinides

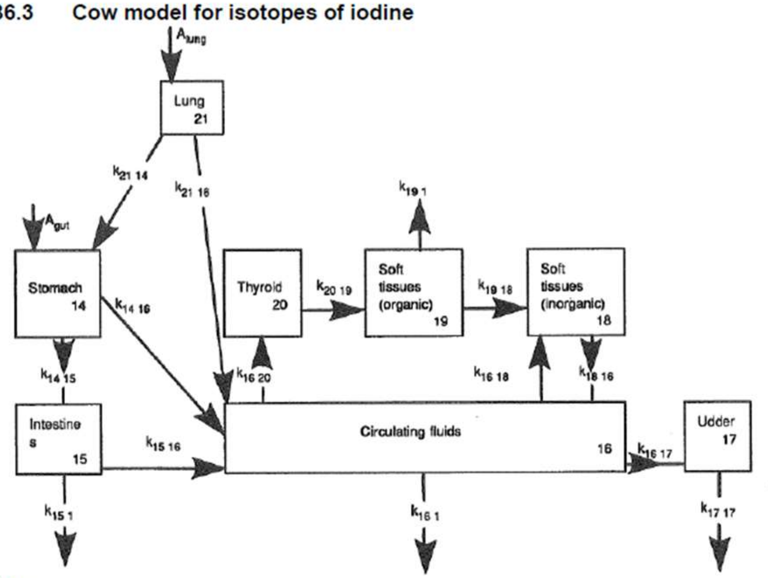
- Cattle (sheep + cows) + undisturbed pasture model
  - Actinides

- No models for tritium or carbon-14

Example Box Model (the FARMLAND model)

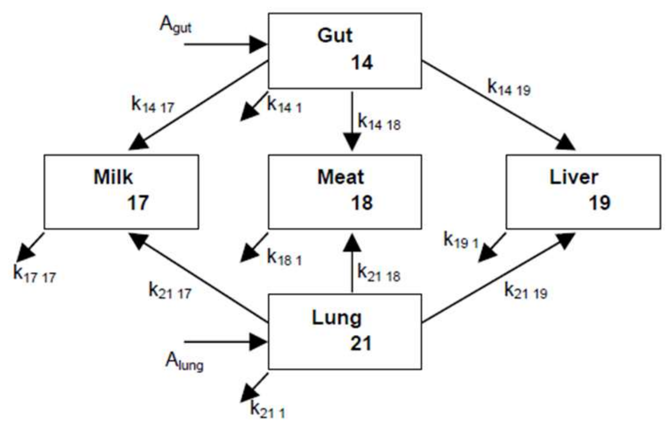
- Cow model for iodine more complex than that for other non actinides and actinides

B6.3 Cow model for isotopes of iodine

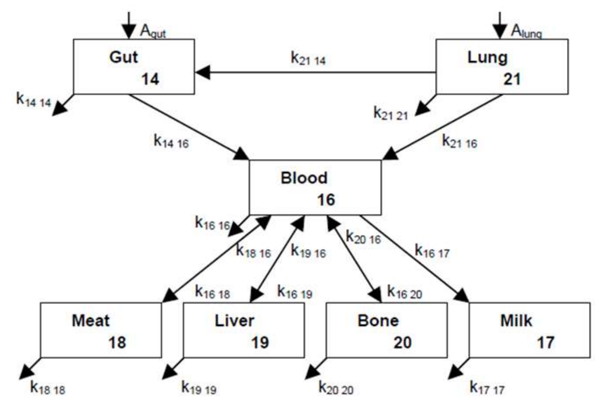


Notes
1 $k_{14\,16}$ represents the early absorption of iodine from the rumen of the cow.
2 The storage of iodine in the soft tissues is represented by two compartments (18 and 19). The organic iodine produced in the thyroid is re-distributed throughout the soft tissues and organs of the body where it remains for some time before being broken down into inorganic iodine.
3 $k_{19\,1}$ represents the excretion of the organic fraction of iodine in the circulating fluids.
4 Periodic slaughter is represented by losses from all compartments, the value of the transfer coefficient is $4.56\ 10^{-4}\ d^{-1}$.

B6.4 Cow model for other non-actinide elements



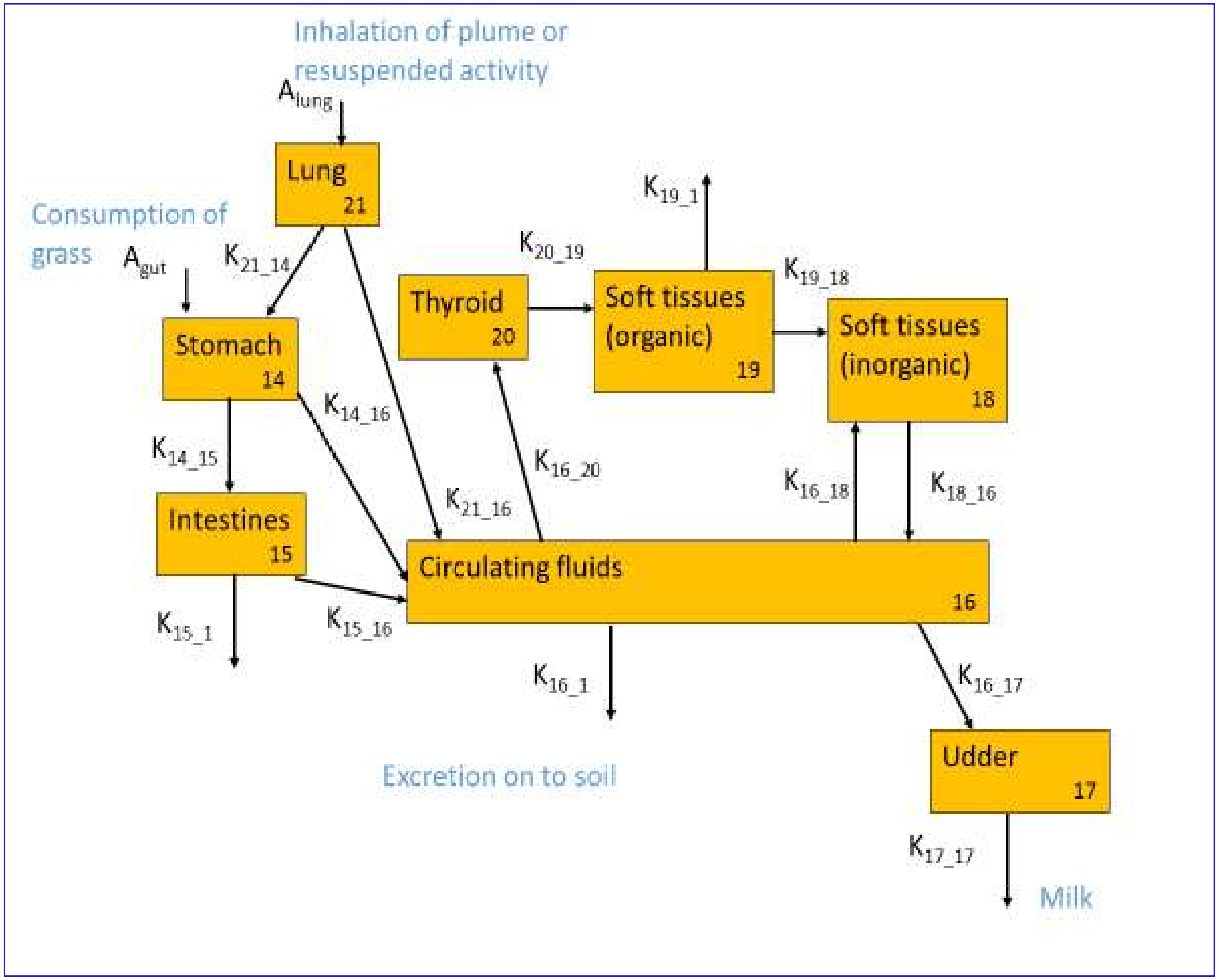
B8 ANIMAL MODEL FOR THE ACTINIDES (CATTLE; SHEEP)



Developing a stand-alone model

Example for iodine in milk

FARMLAND cow-iodine model



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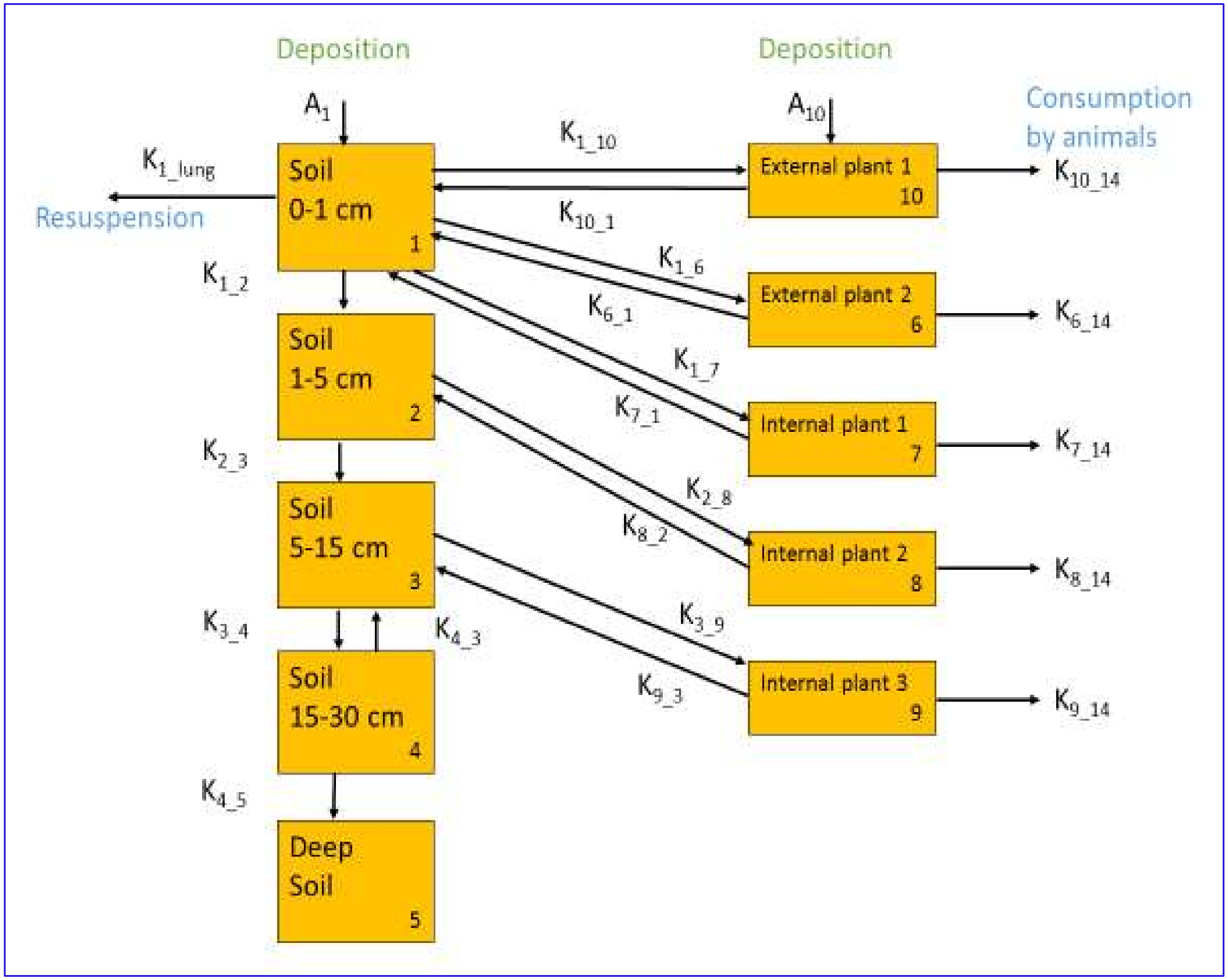
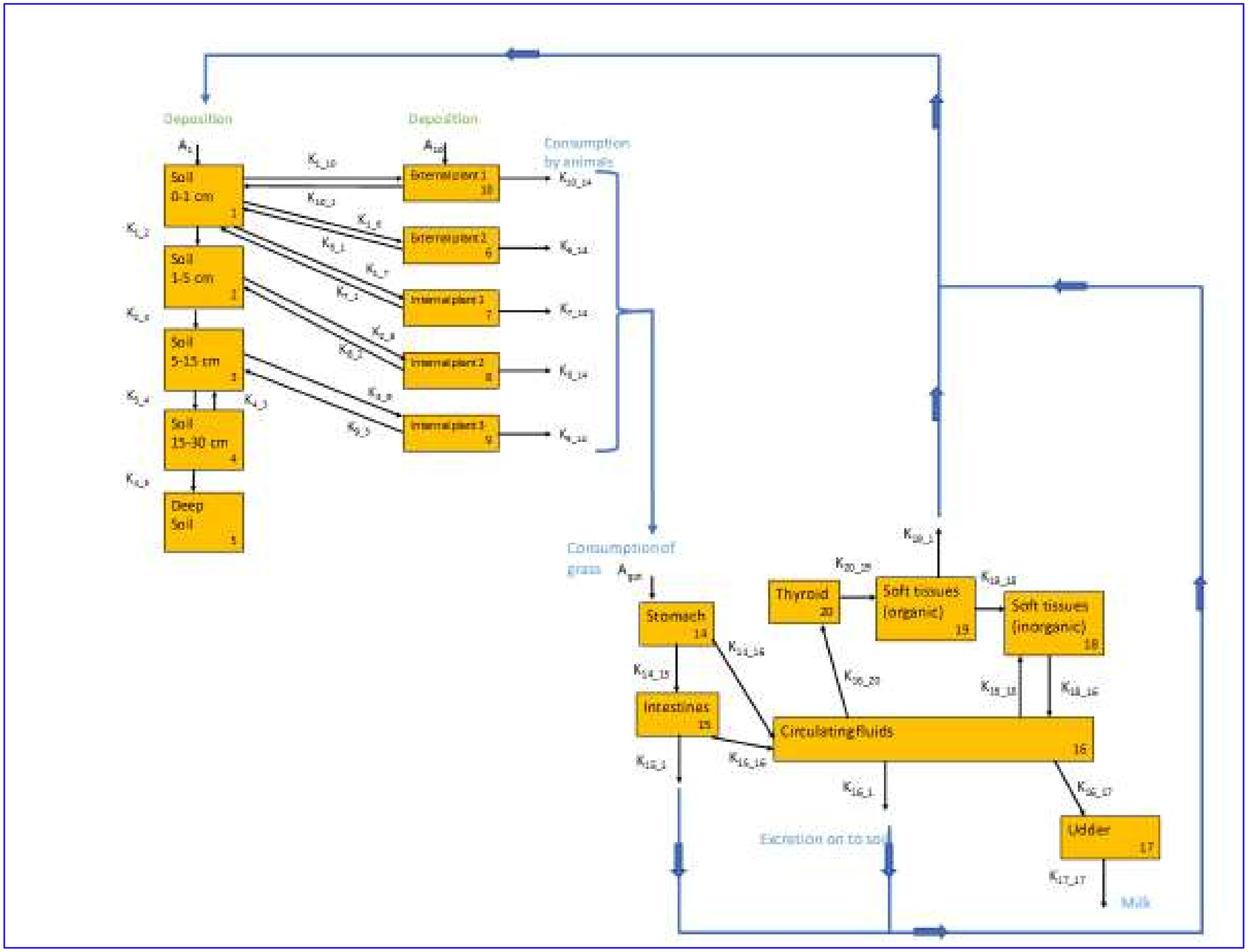
FARMLAND undisturbed pasture model

deposition on soil and grass



FARMLAND combined undisturbed pasture – cow model



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Example problem matrix – 18 boxes gives an 18 x 18 matrix for which the Eigen values and Eigen vectors need to be found

	1	2	3	4	5	6	7	8	9	10	14	15	16	17	18	19	20	milk
1	-8.07E+01	0	0	0	0	8.64E+04	8.64E+04	0	0	0	0	5.00E+00	2.50E+00	0	0	1.20E-01	0	0
2	6.64E-04	-1.45E+01	0	0	0	0	0	8.64E+04	0	0	0	0	0	0	0	0	0	0
3	0	1.72E-04	-5.85E+00	4.03E-06	0	0	0	0	8.64E+04	0	0	0	0	0	0	0	0	0
4	0	0	1.07E-04	-8.65E-02	0	0	0	0	0	0	0	0	0	0	0	0	0	0
5	0	0	0	3.80E-05	-8.64E-02	0	0	0	0	0	0	0	0	0	0	0	0	0
6	2.30E+01	0	0	0	0	-8.64E+04	0	0	0	0	0	0	0	0	0	0	0	0
7	5.76E+01	0	0	0	0	0	-8.64E+04	0	0	0	0	0	0	0	0	0	0	0
8	0	1.44E+01	0	0	0	0	0	-8.64E+04	0	0	0	0	0	0	0	0	0	0
9	0	0	5.76E+00	0	0	0	0	0	-8.64E+04	0	0	0	0	0	0	0	0	0
10	0	0	0	0	0	0	0	0	0	-1.38E-01	0	0	0	0	0	0	0	0
14	0	0	0	0	0	5.20E-02	5.20E-02	5.20E-02	5.20E-02	5.20E-02	-1.28E+00	0	0	0	0	0	0	0
15	0	0	0	0	0	0	0	0	0	0	7.00E-01	-5.89E+00	0	0	0	0	0	0
16	0	0	0	0	0	0	0	0	0	0	4.90E-01	8.00E-01	-1.28E+01	0	2.40E+00	0	0	0
17	0	0	0	0	0	0	0	0	0	0	0	0	3.35E-01	-4.09E+00	0	0	0	0
18	0	0	0	0	0	0	0	0	0	0	0	0	9.19E+00	0	-2.49E+00	5.71E-02	0	0
19	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-2.64E-01	7.34E-02	0
20	0	0	0	0	0	0	0	0	0	0	0	0	7.21E-01	0	0	0	-1.60E-01	0
milk	0	0	0	0	0	0	0	0	0	0	0	0	0	4.00E+00	0	0	0	0

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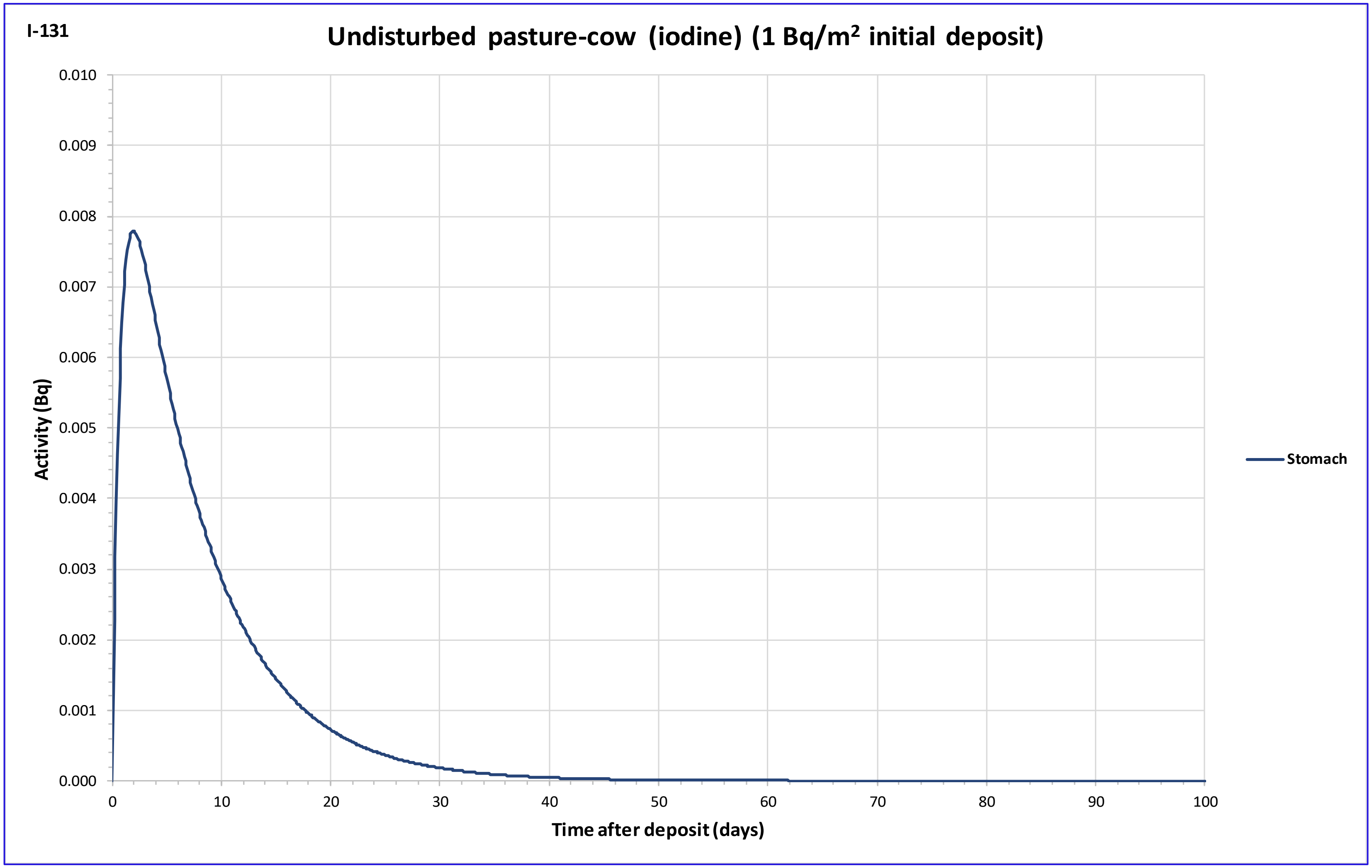
Example of the time-dependent activity in a model compartment

- An example of the time-dependent functions derived is given below for the stomach compartment for a 1 Bq/m^2 of I-131
  - Can be scaled for actual deposited activity

Activity in the stomach compartment as function of time =						
	3.641E-03	x	0.000E+00	x	exp(	0.000E+00	t)
+	1.958E-05	x	0.000E+00	x	exp(	-4.087E+00	t)
+	0.000E+00	x	4.256E-07	x	exp(	-8.641E+04	t)
+	-8.820E-04	x	-4.765E-07	x	exp(	-8.648E+04	t)
+	-2.956E-13	x	-4.255E-07	x	exp(	-8.641E+04	t)
+	0.000E+00	x	-3.106E-15	x	exp(	-8.640E+04	t)
+	3.156E-05	x	4.915E-07	x	exp(	-1.465E+01	t)
+	4.641E-04	x	6.925E-06	x	exp(	-5.887E+00	t)
+	1.501E-02	x	-7.629E-01	x	exp(	-1.277E+00	t)
+	2.193E-02	x	5.010E-05	x	exp(	-6.706E-01	t)
+	1.310E-02	x	-3.310E-05	x	exp(	-2.750E-01	t)
+	-2.136E-01	x	-3.499E-05	x	exp(	-1.517E-01	t)
+	1.678E+00	x	1.867E-05	x	exp(	-8.710E-02	t)
+	-2.675E+00	x	-1.309E-07	x	exp(	-8.646E-02	t)
+	4.930E+00	x	1.795E-06	x	exp(	-8.653E-02	t)
+	3.916E+00	x	-5.128E-08	x	exp(	-8.660E-02	t)
+	0.000E+00	x	0.000E+00	x	exp(	0.000E+00	t)
+	5.062E-01	x	2.252E-02	x	exp(	-1.384E-01	t)
	the 18 constants	The 11th elements of the 18 eigenvectors (the stomach is the 11th box)				the 18 eigenvalues	

Example compartment time-dependent activity



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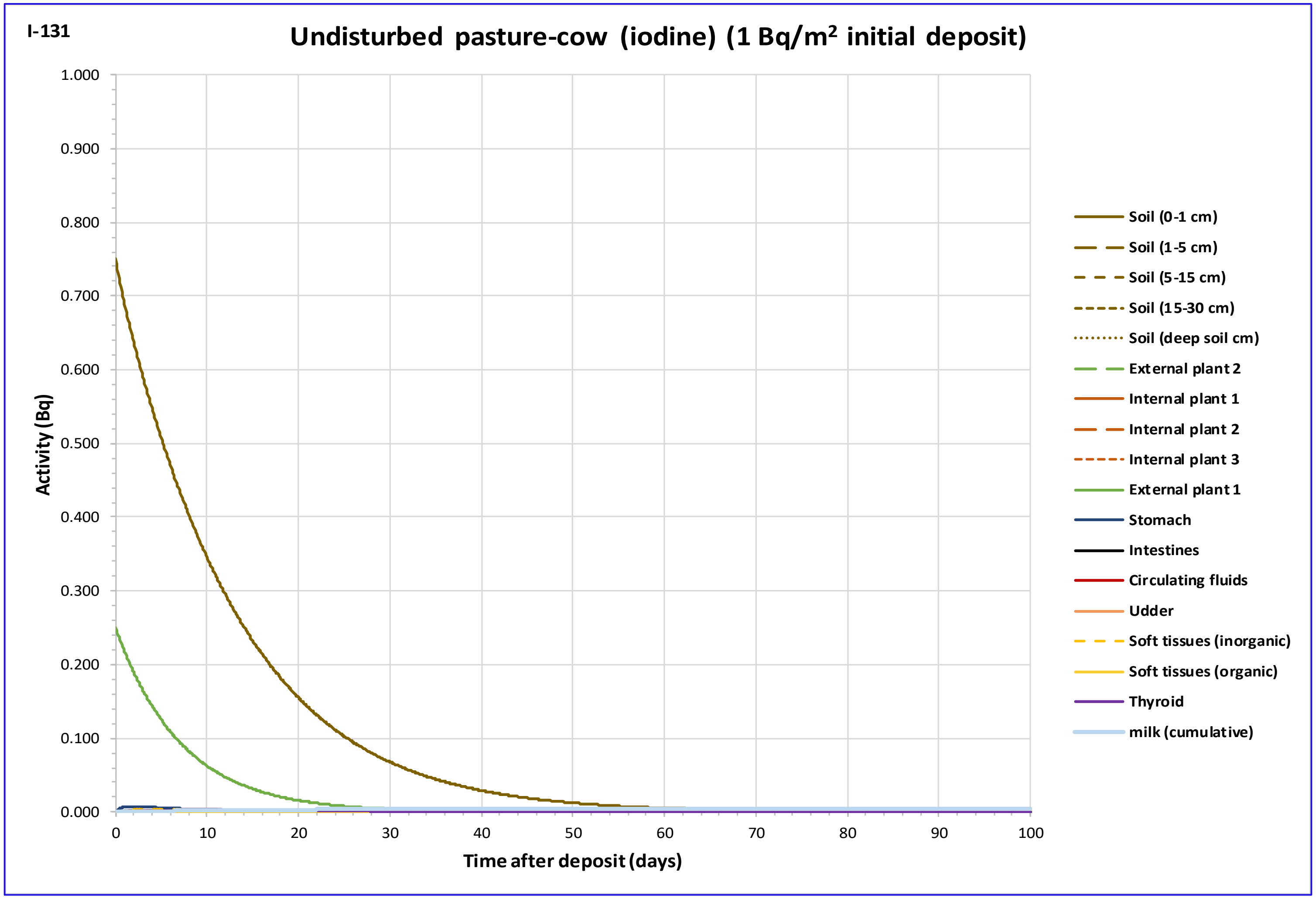
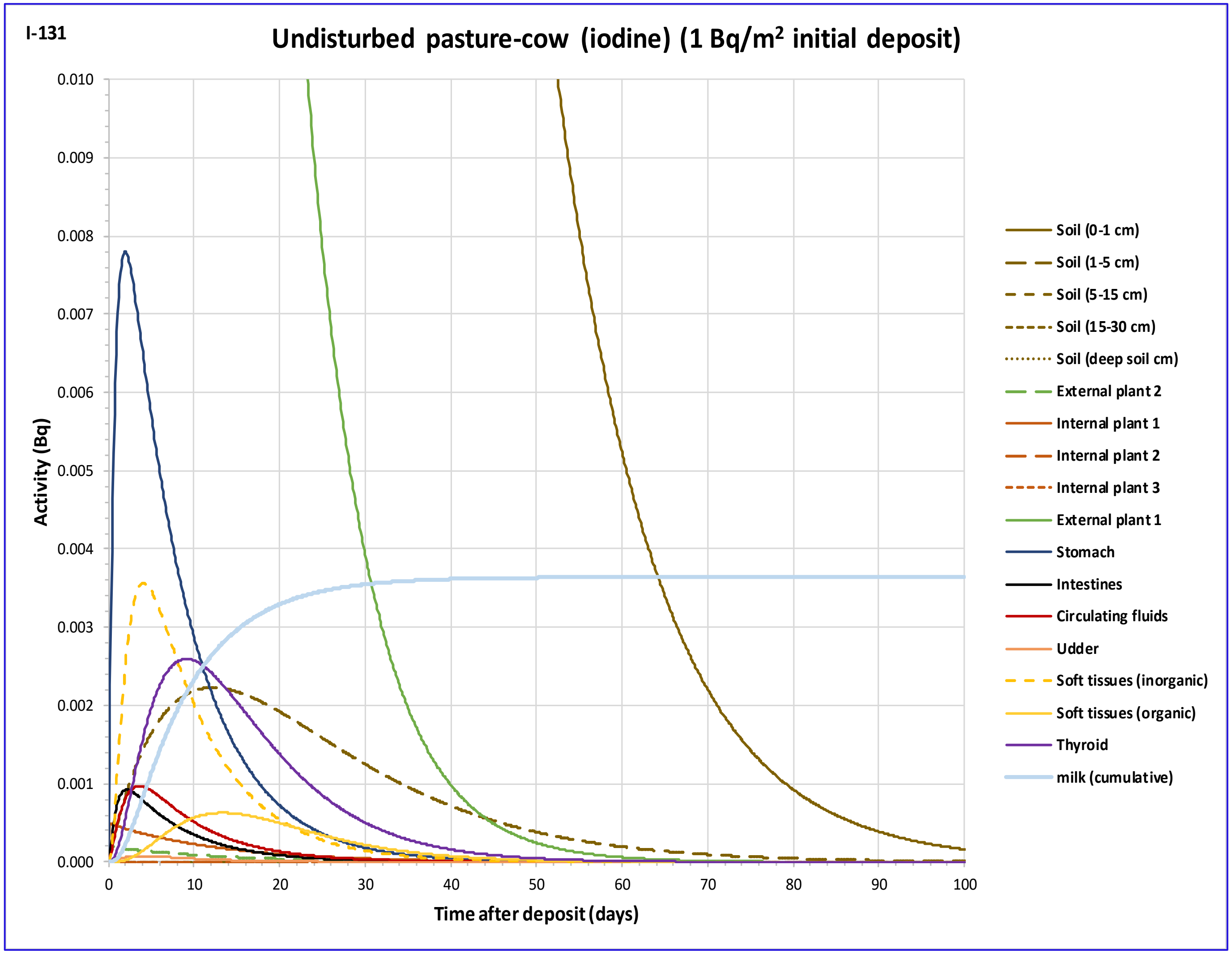
Time-dependent behaviour for all compartments in the model



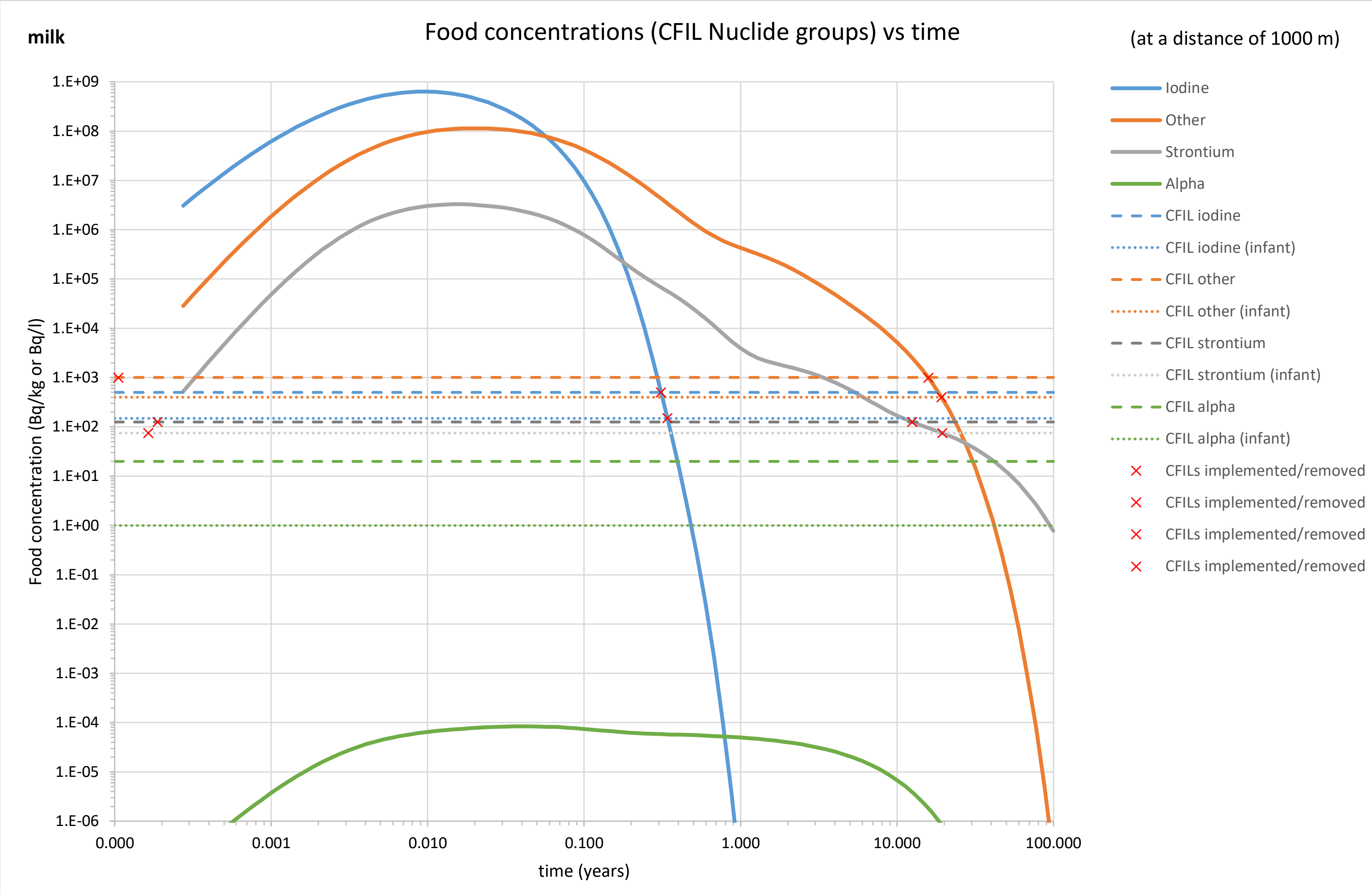
Combined undisturbed pasture - cow model

Time-dependent behaviour for all compartments in the model (expanded scale)



Combined undisturbed pasture - cow model

Example results using the IAEA Fukushima upper estimate source term (atmospheric dispersion with a constant wind direction)



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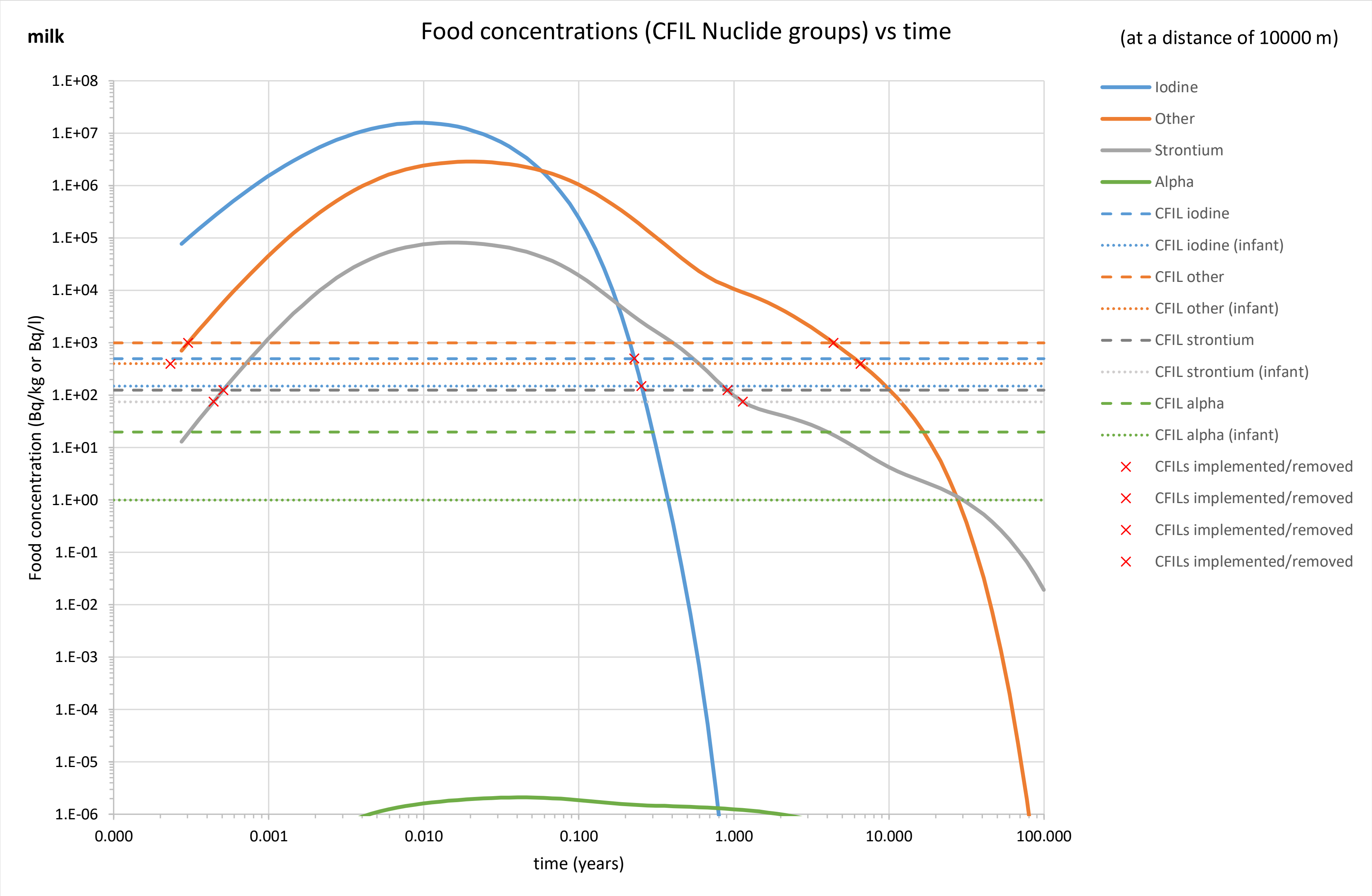

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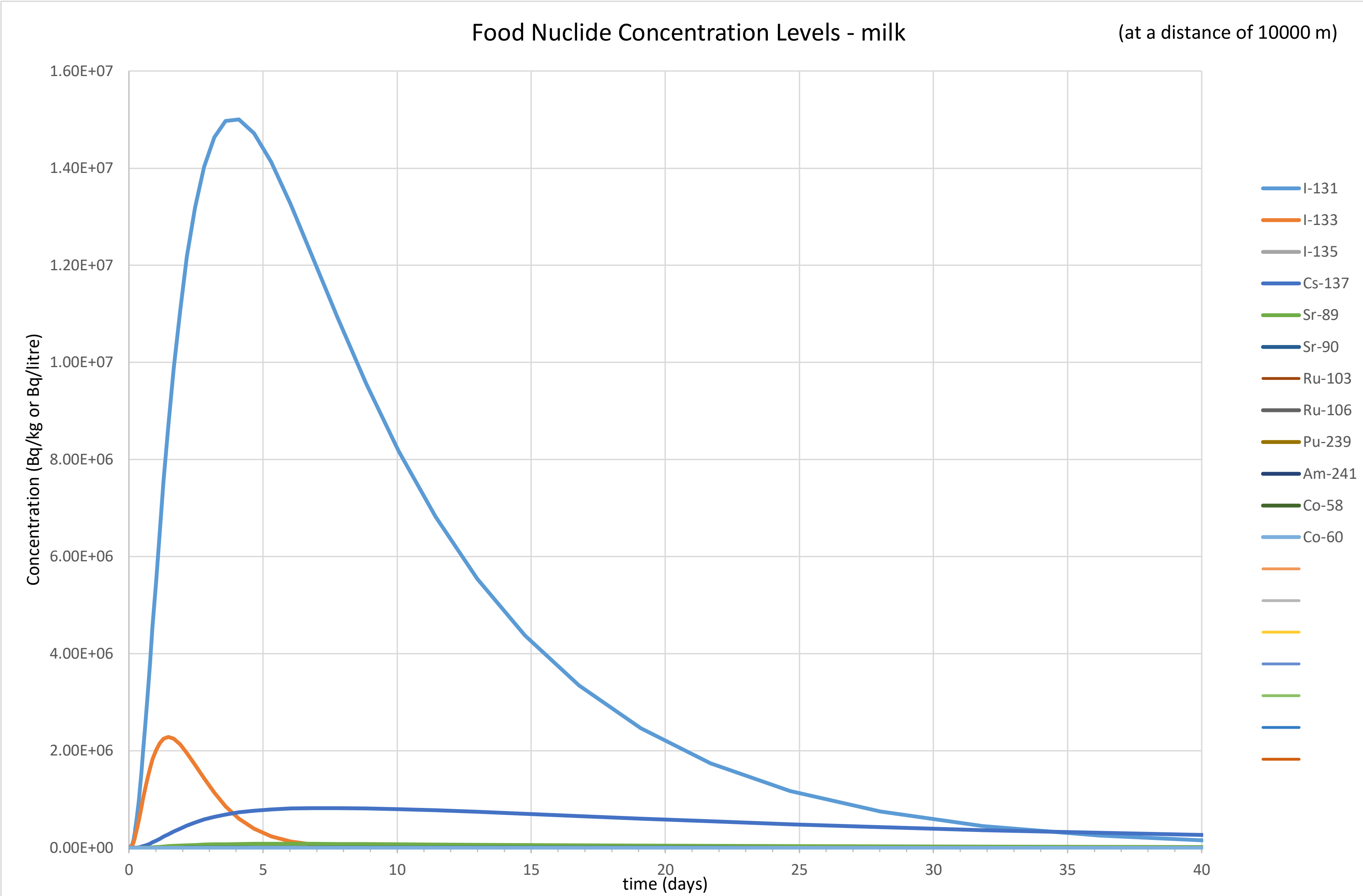

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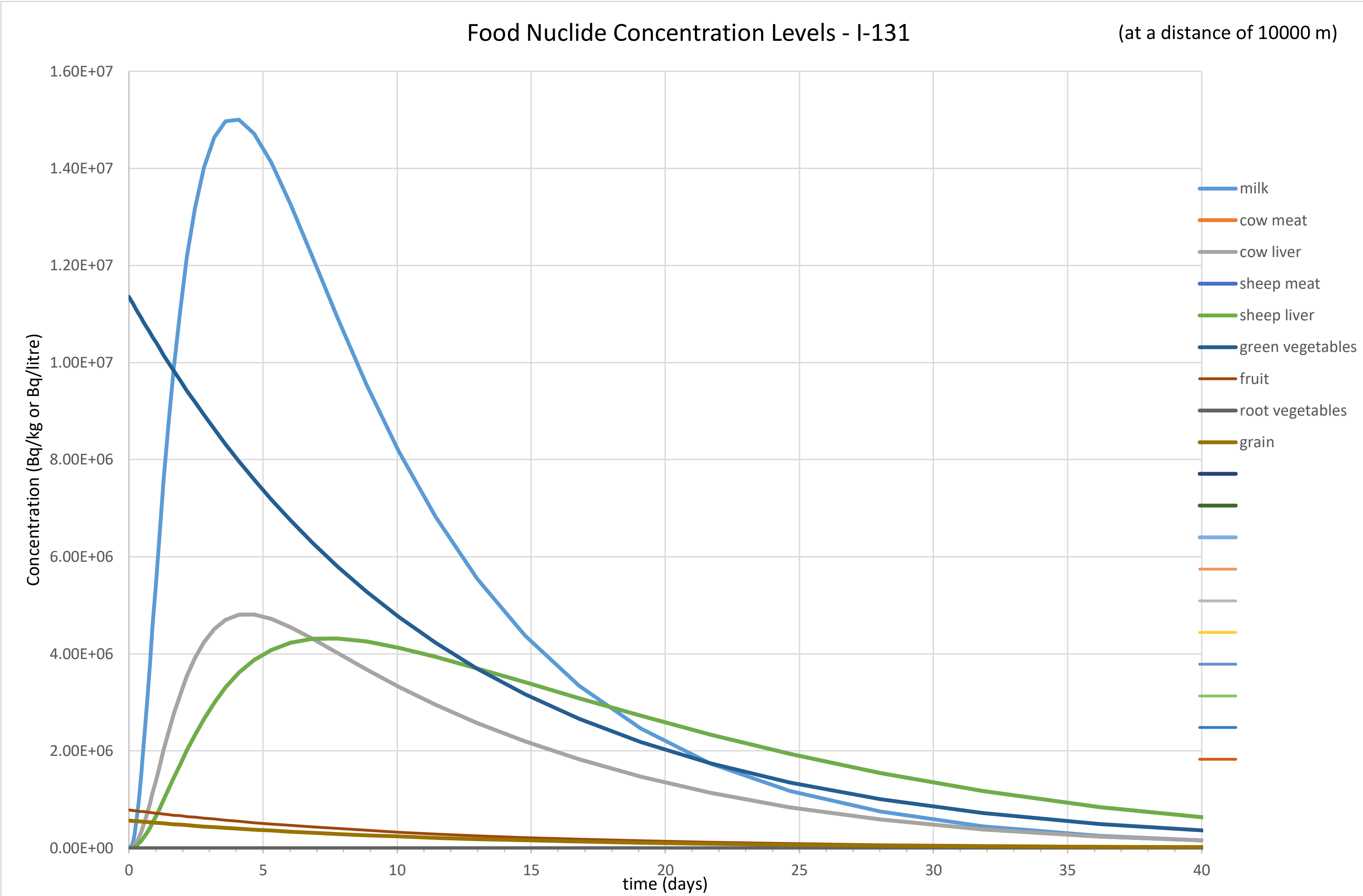

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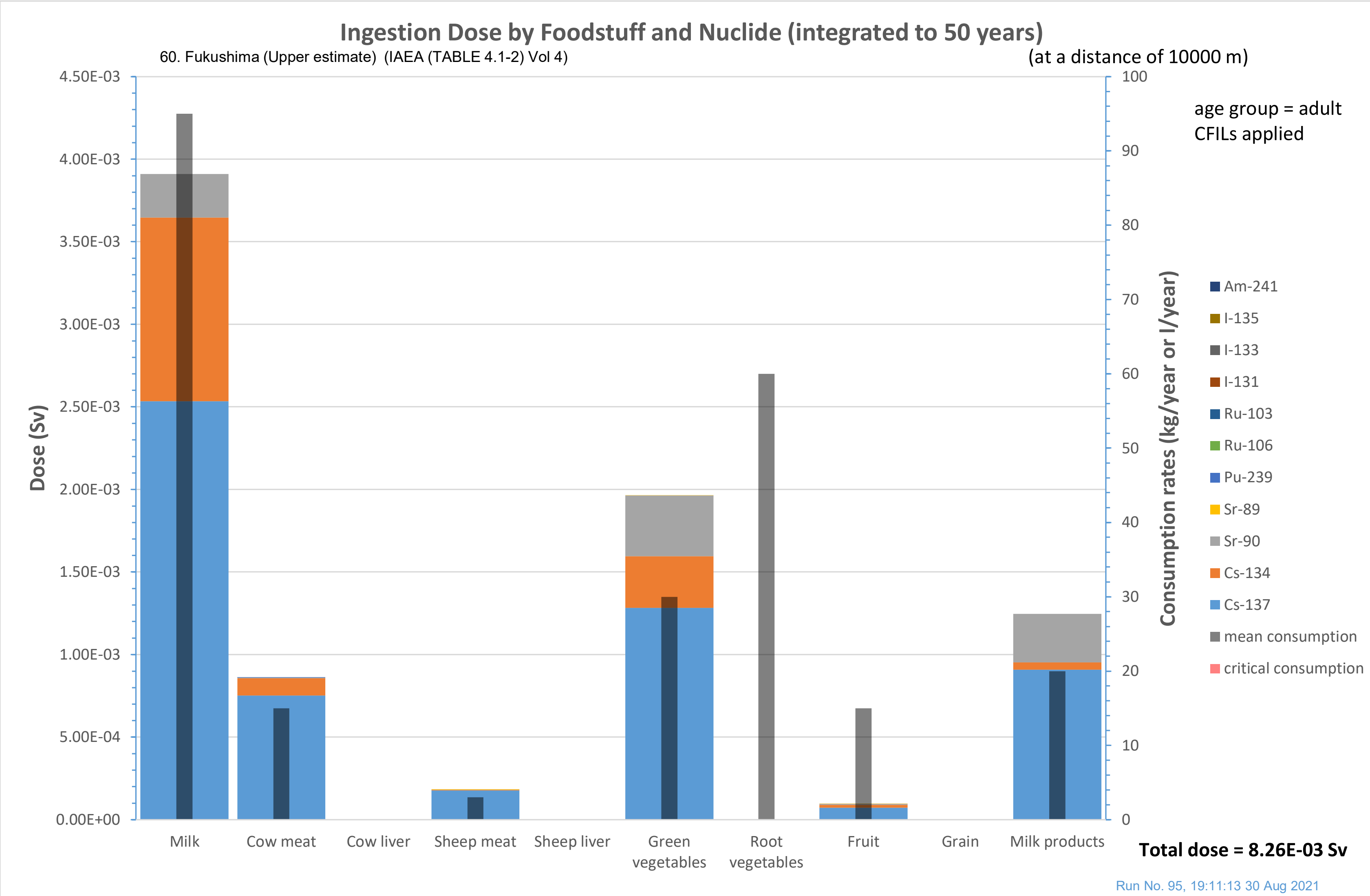

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Summary

Advantages/limitations of an Eigenvalue solution approach

- Limitations
  - Assumes an instantaneous deposit
    - Can't model continuous discharges by this method
- Advantages
  - Time dependence over all time frames can be captured efficiently by a relatively small set of constants (18 x 3 for iodine-milk)
  - Functions are a series of exponential terms and so are easy to differentiate or integrate
    - Can calculate ingestion dose for any integration period
    - Can determine – with relatively simple numerical methods – peak concentrations in food or when concentration reaches a particular value
    - Above method can also be applied to sums of nuclide groups to determine when CFILs (food intervention levels) are reached and durations of CFIL implementation
  - Once Eigenvalues, Eigenvectors, and constants have been determined for unit deposited activity for each nuclide/food group combination, model no longer needed and calculations can be performed using a simple spreadsheet implementation for example scaling for the actual deposited activities
  - Can be used in conjunction with any other model that gives deposited activity for a set of radionuclides
  - Visualizing behaviour in each compartment may allow greater insights into the model

Further information

Transboundary assessments (Article 37 of Euratom Treaty)

COMMISSION RECOMMENDATION

of 11 October 2010

on the application of Article 37 of the Euratom Treaty

(2010/635/Euratom)

6.3. **Evaluation of the radiological consequences of the reference accident(s) and for operations (1) and (2), the accident(s) taken into consideration by the competent authorities for the establishment of the site related national emergency plan**

6.3.1. Accidents entailing releases to atmosphere

Except for operations listed under (1) and (2), if the assessed maximum exposure levels from the reference accident to adults, children and infants in the vicinity of the plant are below 1 mSv and there are no exceptional pathways of exposure, e.g. involving the export of foodstuffs, no data on exposure levels in other affected Member States are required if exposure levels in the vicinity of the plant are provided.

— assumptions used to calculate the releases to atmosphere,

— release paths; time patterns of the releases,

— amounts and physico-chemical forms of those radionuclides released which are significant from the point of view of health,

— models and parameter values used to calculate for the releases their atmospheric dispersion, ground deposition, re-suspension and transfer via food chains and to evaluate the maximum exposure levels via the significant exposure pathways in the vicinity of the plant and for other affected Member States,

— maximum time-integrated concentrations of radioactivity in the atmosphere near the ground and maximum surface contamination levels (in dry and wet weather) for the most exposed areas in the vicinity of the plant and for relevant areas in other affected Member States,

— expected levels of radioactive contamination of foodstuffs which might be exported to other affected Member States,

— corresponding maximum exposure levels: effective dose to adults, children and infants living in the vicinity of the plant and in relevant areas of other affected Member States taking account of all significant exposure pathways.

https://eur-lex.europa.eu/legal-content/EN/TXT/?uri=CELEX%3A32010H0635

Council Food Intervention Levels (CFILS)

COUNCIL REGULATION (Euratom) 2016/52

of 15 January 2016

laying down maximum permitted levels of radioactive contamination of food and feed following a nuclear accident or any other case of radiological emergency, and repealing Regulation (Euratom) No 3954/87 and Commission Regulations (Euratom) No 944/89 and (Euratom) No 770/90

MAXIMUM PERMITTED LEVELS OF RADIOACTIVE CONTAMINATION OF FOOD

The maximum permitted levels to be applied to food shall not exceed the following:

Isotope group/Food group	Food (Bq/kg) (1)			
	Infant food (2)	Dairy produce (3)	Other food except minor food (4)	Liquid food (5)
Sum of isotopes of strontium, notably Sr-90	75	125	750	125
Sum of isotopes of iodine, notably I-131	150	500	2 000	500
Sum of alpha-emitting isotopes of plutonium and transplutonium elements, notably Pu-239 and Am-241	1	20	80	20
Sum of all other nuclides of half-life greater than 10 days, notably Cs-134 and Cs-137 (6)	400	1 000	1 250	1 000

(1) The level applicable to concentrated or dried products is calculated on the basis of the reconstituted product as ready for consumption. Member States may make recommendations concerning the diluting conditions in order to ensure that the maximum permitted levels laid down in this Regulation are observed.
(2) Infant food is defined as food intended for the feeding of infants during the first 12 months of life which meets, in itself, the nutritional requirements of this category of persons and is put up for retail sale in packages which are clearly identified and labelled as such.
(3) Dairy produce is defined as products falling within the following CN codes including, where appropriate, any adjustments which might subsequently be made to them: 0401 and 0402 (except 0402 29 11).
(4) Minor food and the corresponding levels to be applied to them are set out in Annex II.
(5) Liquid food is defined as products falling within heading 2009 and Chapter 22 of the Combined Nomenclature. Values are calculated taking into account consumption of tap-water and the same values could be applied to drinking water supplies at the discretion of competent authorities in Member States.
(6) Carbon-14, tritium and potassium-40 are not included in this group.

Levels based on a reference level of 1 mSv per year from consuming 10% contaminated food

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IAEA Fukushima estimated source term

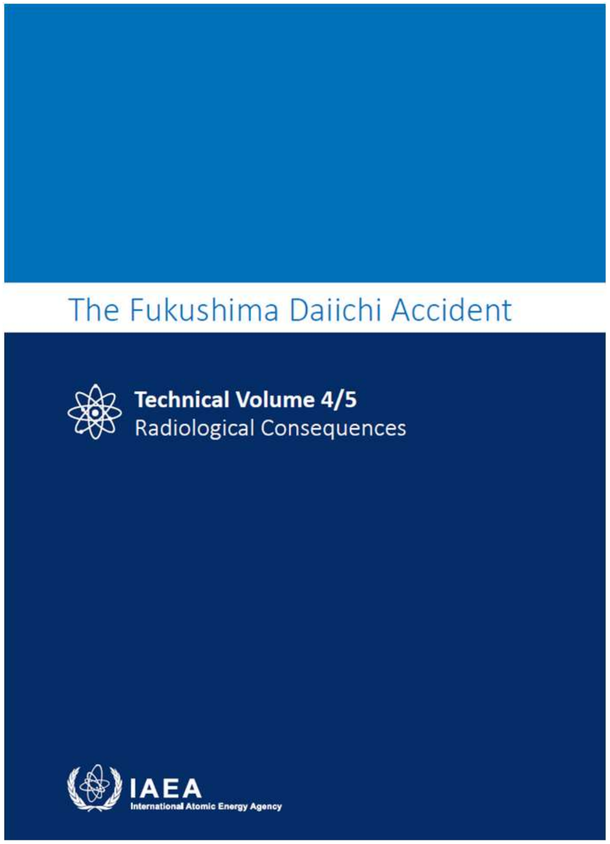

TABLE 4.1–2. AN ILLUSTRATIVE RANGE OF ESTIMATES OF ATMOSPHERIC RELEASES OF A WIDER RANGE OF RADIONUCLIDES (PBq) AND COMPARISON WITH THOSE FROM THE CHERNOBYL ACCIDENT

Radionuclide	Fukushima Daiichi[a]	Chernobyl[b]
Fission noble gases		
Kr-85	6.4–32.6	33
Xe-133	6 000–12 000	6 500
Volatile fission products		
Te-129m	3.3–12.2	240
Te-132	0.76–162	~1.15×10^3
I-131	100–400	~1.76×10^3
I-133	0.68–300	2500
Cs-134	8.3–50	~47
Cs-136	—	36
Cs-137	7–20	~85
Semi- and low volatile fission products		
Sr-89	4.3×10^{-2}–13	~115
Sr-90	3.3×10^{-3}–0.14	~10
Ru-103	7.5×10^{-6}–7.1×10^{-5}	>168
Ru-106	2.1×10^{-6}	>73
Ba-140	1.1–20	240
Refractory elements		
Zr-95	0.017	84
Mo-99	8.80×10^{-8}	>72
Ce-141	0.018	84
Ce-144	0.011	~50
Np-239	0.076	400
Pu-238	2.4×10^{-6}–1.9×10^{-5}	0.015
Pu-239	4.1×10^{-7}–3.2×10^{-6}	0.013
Pu-240	5.1×10^{-7}–3.2×10^{-6}	0.018
Pu-241	3.3×10^{-7}–1.2×10^{-3}	~2.6
Cm-242	9.8×10^{-6}–10^{-4}	~0.4

[a] Range of estimates from JNES, 2012 [19], NISA, 2011 [20, 21], IRSN-2, 2012 [22–24], IBRAE, 2012 [25–27], with the exception of Xe-133, I-131 and Cs-137, where the estimated range is based on the greater number of estimates described in Technical Volume 1, Section 1.4 (excluding early estimates, made in March–April 2011).
[b] From Ref. [28].

Thank you

Jacobs

Challenging today.
Reinventing tomorrow.